Chapter 4: Conclusions

Accumulators and Object Files: Compatible hypotheses?

The evidence presented in Chapter 3 suggests that nonhuman primates use both Accumulator-like mental magnitudes as well Object Files to spontaneously represent numbers. Table 4.1 summarizes the results of these experiments. Experiments 3.1 – 3.4 demonstrate that monkeys can discriminate between arithmetical outcomes of 4 and 8, though they cannot discriminate between outcomes of 4 and 6 for similar operations. This pattern of results is consistent with the Weber-fraction limit upon discrimination predicted by the Accumulator model of numerical representation; as numbers become farther apart, the large ratio distinguishing them allows the Accumulator system to resolve the different representations of each number in the face of the noise inherent in this system. These four experiments are the first to demonstrate that nonhuman animals spontaneously represent large numbers, and that they do so using Accumulator-like mental magnitudes.

Table 4.1. Summary of Results Reported in Chapter 3

Operation	Pass/Fail	Experiment
3+1 = 4 v. 8	Pass	3.1
2+2 = 4 v. 8	Pass	3.2
4+4 = 4 v. 8	Pass	3.3
2+2 = 4 v. 6	Fail	3.4
1YDB + 1YDB = 2 v. 3	Pass	3.5
1CDB + 1CDB = 2 v. 3	Fail	3.6
4+1 = 4 v. 5	Fail	3.7
4+1 = 5 v. 8	Pass	3.7

(Legend: YDB = yellow dumbbell; CDB = clear dumbbell)

Experiments 3.5 and 3.6 demonstrate that although monkeys are unable to discriminate between 4 and 6 objects as outcomes of arithmetical operations, they can still discriminate between 2 and 3 objects. Because these outcomes differ by the same ratio, these results indicate that performance with small numbers is attributable to the workings of a system specialized for numbers in this limited range. Namely, the Object File system predicts that, regardless of ratio, monkeys should discriminate between all comparisons among numbers in the range of 1-4. This pattern of behavior is found consistently in looking time experiments (Chapter 3; Hauser, MacNeilage, and Ware, 1996; Hauser and Carey, in preparation) as well as search experiments (Hauser, Carey, and Hauser, 2000). Experiments 3.5 and 3.6 also demonstrate that the rules governing the assigning of Object Files are similar in nonhuman primates and humans. Clear dumbbells were not assigned with Object Files, though visible yellow dumbbells were. Thus, the monkeys were forced to enumerate individual lemons in Experiment 3.6, though they could enumerate individual dumbbells in Experiment 3.5. Scholl, Pylyshyn and Feldman (2001) found similar object enumeration using multiple object tracking experiments with human adults; the visual system of humans, like that of rhesus monkeys, only considers dumbbells as objects when they possess rigid, continuous, and visible connections.

Finally, in Experiment 3.7, monkeys did not discriminate between an impossible outcome of 4+1 = 4 and a possible outcome of 4+1 = 5, though they did discriminate between this possible outcome and an impossible outcome of 4+1 = 8. Coupled with the results of Hauser and Carey (in preparation) demonstrating that monkeys do discriminate between an impossible outcome of 2+1 = 4 and a possible outcome of 2+1 = 3, my results suggest that parallel individuation, and therefore, the use of Object Files for enumeration, is limited to about 4 objects. This finding also replicates those of Hauser, Carey, and Hauser (2000) indicating that, in a search experiment, monkeys can discriminate between 1 vs. 2, 2 vs. 3, and 3 vs. 4, but not 4 vs. 5. The fact that subjects in Experiment 3.7 found a 4+1 = 8 event unexpected suggests that the Accumulator system can resolve differences between numbers that differ by a ratio smaller than 1:2. Given all the results reported in Chapter 3, the limits of this system lies in between a 2:3 ratio and 5:8 ratio.

Despite an inability to use natural language, then, it appears that nonhuman primates spontaneously use two complementary systems to represent numbers: an Object File system for small numbers and an Accumulator system for large numbers. Evidence from a variety of paradigms suggests that adult humans use these systems as well (Gallistel, and Gelman, 2000; Whalen, Gallistel, and Gelman, 1999; Trick and Pylyshyn, 1994; Khaneman, Treisman, and Gibbs, 1992), particularly in situations where task demands constrain the use of language. Consequently, current evidence supports the hypothesis that some formats of numerical representation do not require the use of language, or even, a mind that will develop language. Instead, few human mathematical capacities are, in fact, unique to humans. Our evolutionarily ancient mental toolkit in the domain of number includes both the capacity to precisely enumerate small numbers of objects and the capacity to approximately enumerate large numbers of objects. In the number domain, language buys no more than the ability to enumerate large numbers precisely. No current evidence demonstrates that animals share our ability to precisely distinguish between two large numbers. It would appear, though, that specifically this unique capacity holds the key to the many branches of abstract mathematics and physics that humans have invented in recent phylogenetic history.

Controlling for continuous extent: A potential confound

A serious confound in these experiments, as well as in other studies investigating the numerical representations of human infants and monkeys, is the potential that individuals track a continuous variable different from, but correlated with, number. This is particularly worrisome given that the Weber-fraction limit that is taken as evidence for analog magnitude representations of number may also apply to the discrimination of other continuous variables. In other words, other continuous variables are probably represented in an analog magnitude format as well. Consequently, do the results of the experiments in Chapter 3 necessarily demonstrate that monkeys represent number, as opposed to other properties of the displays?

The literature on spontaneous number representations has only recently started to investigate this potential confound. Hauser and Carey (in preparation) and Uller, Hauser, and Carey (2001) demonstrated that two primate species discriminate between an impossible outcome of 1+1 = 1 large object, and a possible outcome of 1+1 = 2, suggesting that monkeys do generate an expectation of number, and not of an overall volume or mass. Unfortunately, however, mass and volume are not the only potential confounding variables.

This point is well taken considering recent habituation experiments with human infants. Researchers have found that infants dishabituate more to changes in a continuous variable than changes in number when number is pitted against contour length (Clearfield and Mix, 1999) or surface area (Feigenson, Carey, and Spelke, 2002),. These data are especially troubling because they pertain to both discriminations among small numbers, and discriminations among numbers that differ by a large ratio. When continuous variables were controlled across trials, infants failed to discriminate sets of 1 and 2, as well as sets of 2 and 3 (Clearfield and Mix, 1999; Feigenson, Carey, and Spelke, 2002). These data do not support the hypothesis that infants perform a correspondence operation over Object Files, nor do they support the hypothesis that infants use magnitude representations of *number* because this system predicts success in discriminations of 1 and 2 objects. Feigenson, Carey, and Spelke (2002) have shown a similar pattern in 1+1 and 2-1 expectancy violation experiments with outcomes of 1 and 2. A problem with this experiment, however, is that it only compared outcomes of impossible continuous extent / possible number with outcomes of impossible number / possible continuous extent. A demonstration that infants do not look reliably longer at an outcome of impossible number and possible continuous extent compared with an outcome of possible number and possible continuous extent is necessary to conclude that they do not represent number at all.

In contrast to these experiments, two pieces of data suggest that infants do actually spontaneously represent numbers, at least in some sense. Controlling for all relevant variables, Xu and Spelke (2000) found that infants dishabituate during a change from 8 to 16 dots and vice-versa, though they do not dishabituate to changes between 8 and 12 dots. Xu and Spelke have recently extended this work and demonstrated that infants spontaneously detect differences between 16 and 32 dots, as well (Xu, 2000), confirming the hypothesis that infants spontaneously use analog magnitudes to represent number. These data are extremely puzzling, though, given infants' failure to discriminate 1 and 2 objects when surface area is controlled (Feigenson, Carey, and Spelke, 2002). Hence, it appears that infants only use magnitude representations when discriminating large numbers. Many more experiments are required to understand the mechanism by which these representations are spontaneously deployed in one context – for numbers as big or bigger than 8 – but not in another context – for small numbers such as 1 and 2.

A second piece of evidence more indirectly suggests that infants represent number. Feigenson and colleagues (Feigenson, Carey, and Hauser, 2002; Feigenson, Carey, and Spelke) made the observation that though infants seem to track continuous variables and not number in habituation experiments, they, nevertheless, have serious trouble discriminating among sets of more than 3 or 4 objects. In other words, experiments in which infants demonstrate sensitivity to continuous variables and not number still demonstrate the set-size signature typical of Object File representations. Starkey and Cooper (1980), for example, found that infants discriminate 2 and 3 objects, but not 4 and 6 objects. Similarly, Feigenson, Carey, and Hauser (2002) found that infants reliably choose a box with 2 pieces of food more often than they choose a box with 1 piece of food, though they are at chance when faced with a choice between 3 and 6 pieces. In this experiment, infants were also at chance when amount of edible food was equated across conditions of 1 versus 2 pieces, suggesting that here, as in other experiments, infants track continuous extent and not number. If this is the case, why, then, are these discriminations constrained by a set-size limit? Tracking a continuous extent should be limited by the ratios between extents, and should have nothing to do with number of objects.

These observations lead Feigenson and her colleagues to the following hypothesis: Infants must represent objects in these various experiments with Object Files, but they compare these representations along a continuous dimension stored in these files. For example, infants will construct an Object File representation of a Mickey Mouse doll on a stage and store the surface area of this doll in its respective Object File. When a second doll is added behind an occluder, they do the same. Finally, when they view the unexpected outcome, rather then perform a correspondence operation between the dolls on the stage and the Object Files open, they compare the total surface area stored in these files with the surface area present on the stage. This hypothesis requires an Object File in order represent an object's continuous extent. Consequently, comparisons along this dimension can only be computed for object*s* numbering in a range for which Object File*s* are available. Infants fail to discriminate between 4 and 6 because they do not possess enough Object Files to represent this many objects, and, as a result they cannot represent information about the continuous extent of more than four objects. In the case of 2 and 3, though, the availability of Object Files provides vessels, if you will, for the storage if information about continuous extent. Therefore, infants do discriminate between these numbers. In this sense, although infants do not discriminate based on number in habituation, expectancy violation, or choice experiments with small sets, they have number represented in so far as they have opened a discrete Object File for every object in these displays. As Feigenson, Carey, and Hauser (2002) put it, "The object-file representations maintain numerical information implicitly" (pp. 155).

This analysis raises two questions with respect to the data presented in Chapter 3: (1) can these data be explained exclusively by mechanisms that track continuous extent, and not number? And (2) can these data be explained by the hypothesis put forward by Feigenson and colleagues (Feigenson, Carey, and Hauser, 2002; Feigenson, Carey, and Spelke, 2002) to explain experiments with infants?

The data in Chapter 3 cannot be explained exclusively in terms of a mechanism used to track continuous extent. The extent of all continuous variables in possible and impossible conditions of Experiment 3.5 (the yellow dumbbells) is identical to the extent of all continuous variables in the respective possible and impossible conditions of Experiment 3.6 (the clear dumbbells). Nevertheless, monkeys discriminate between possible and impossible outcomes in Experiment 3.5 and not in Experiment 3.6. This difference cannot be a consequence of a difference between ratios in continuous extent, as no such difference exists, and therefore, it must be a consequence of the absolute number of objects in the two experiments. The 2 or 3 objects in the different conditions of Experiment 3.5 fall within the range of the Object File system, while the 4 or 6 objects in Experiment 3.6, from the perspective of the visual system's tracking mechanisms, fall outside the Object File range.

However, Feigenson's hypothesis can explain this particular result. In fact, it fits with it quite nicely. Though the continuous extent in these experiments is equated, the large numbers of objects in the clear dumbbells experiment over taxes the Object File system. There is no place, then, to store information about the continuous extent differences in the impossible and possible conditions, though if this information were stored, a successful discrimination could be made. In the yellow dumbbell experiment, however, the small number of objects allows storage in Object Files of the same continuous extent information that could not be stored in the clear dumbbells experiment.

This information is then used to make the discrimination between possible and impossible outcomes observed in Experiment 3.5, and the monkeys do not use a correspondence operation between objects and Object Files to make this discrimination. This, at least, is how this hypothesis would explain these data. However, experiments like those of Feigenson, Carey, and Spelke (2002) that pit number against continuous extent should be conducted with these monkeys before one can clearly distinguish between these different hypotheses. Monkeys may well succeed at discriminating based on number when continuous extent is controlled. I will address a possible reason for such a divergence between monkeys and infants below.

What about the rest of the data? Are the discriminations between 4 and 8 explainable by an exclusively continuous extent account, and/or by Feigenson's account? Monkeys' successful discrimination of outcomes of 4 and 8 in Experiments 3.1 – 3.3 are not explainable by Feigenson's hypothesis. This is because these discriminations occur despite these numbers standing outside the Object File range. If monkeys discriminate outcomes in these experiments using continuous extent, it is not by storing extent information in Object Files because not enough Object Files are available for this purpose. Therefore, in contrast to the infant case, if monkeys track continuous extent in these experiments, they must do so without the use of Object Files. Further experiments should ascertain whether monkeys in these experiments fail, as infants do, when continuous extent is controlled. However, if monkeys do discriminate between 4 and 8 based on continuous extent in Experiments 3.1 – 3.3, it is hard to imagine why they would not discriminate between 4 and 6 in Experiment 3.4. By Feigenson's hypothesis, this failure is because they lack Object Files to store information about these, otherwise, discriminable extents. In the case of 4 and 8 though, they also lack enough Object Files for use in the way described by this theory; yet they make the discrimination. We are left to conclude, then, that the Weber constraints on discriminable continuous extent ratios change depending on whether or not Object Files are in use. It is unclear why such a difference would exist.

It seems more likely that monkeys may fail to discriminate small numbers when continuous extent and number are pitted against one another, though the results with 4 and 8 will hold in this situation. This result, I think, would be like the results of Xu and Spelke (2000) demonstrating that, when continuous extent is controlled, infants use magnitude representations to discriminate relatively very large numbers. I have two reasons to believe this. First, an extensive history of training experiments with animals, which I discussed in Chapter 1, demonstrates that when continuous extent is controlled, animals use Accumulator-like magnitude representations of number. If animals can learn to use these representations – indeed, if in experiments such as that of Brannon and Terrace (1998) they spontaneously generalize the use of these representations after training -- I see no reason why they would not use these representations spontaneously. This issue will not be resolved, however, until experiments like those of Feigenson, Carey, and Spelke (2002) are run with monkeys.

Second, there is good reason to expect a difference between monkeys and infants with respect to the use of magnitude representations. Specifically, infants have to relate their nonlinguistic representations of number to the number words that they eventually learn when they begin to speak. This already complex computational problem would become even harder for infants if they had to relate words to not one, but two representational number formats! If the Object File system is the way infants primarily represent small numbers, then it might be worthwhile for infants to avoid using (or otherwise suppress) magnitude number representations when they begin to represent numbers linguistically. More experiments are required to determine if magnitude representations of number do in fact emerge later in life, and, perhaps, are available very early on before word learning sets in. Further, experiments are required to determine if monkeys are really different from infants in the way that I propose they are; namely, their magnitude representations of number, as well as Object File ones, are available throughout their lifetime because monkeys never face problems related with word learning.

To summarize what I have said about continuous extent, to date no clear conclusion can be drawn from the data that I presented. The successful discrimination of yellow dumbbells in Experiment 3.5 certainly cannot be explained exclusively in terms of continuous extent, though a combination of Object Files and continuous extent may explain this result. The discriminations between 4 and 8 and between 5 and 8 in the other experiments may be explainable in terms of continuous extents, but again, if it is the case that monkeys behave like infants in experiments with small numbers that pit number against extent, then the only explanation of all the data in Chapter 3 will be by appeal to a change with respect to what is a discriminable extent ratio when Object Files are available or not. Overall, these data, as well as those from infants leave many empirical questions open. Of note, I think, future research will need to explain the mechanism by which magnitude representations suddenly become engaged for very large numbers but not for small ones. Do differences exist in the use of the two number systems between monkeys and infants, and if so, why? And, if Feigenson's theory is correct, and infants, and perhaps monkeys, have Object Files active as they make discriminations based on continuous extent, why do they not use this implicitly available numerical information to make discriminations based on number as well?

Future directions: The ecology and biology of guesstimating

In this thesis, I have argued that nonhuman primates spontaneously represent large numbers approximately, and that they spontaneously represent small numbers precisely. If replications and appropriate controls for continuous extent converge with the results I have presented and confirm this argument, then we will have established a good foundation to begin asking more sophisticated questions about numerical representation in animals as well as humans. In particular, experiments like those presented here have paved the road to ask more interesting questions about the neural basis of numerical cognition as well as the natural contexts in which animals represent numbers.

I argued in Chapter 1 that animals must face problems that require an understanding of numbers and quantities on a regular basis. While we can easily speculate about the various contexts in which this claim might be true, it is somewhat harder to find cases of animals representing number in the wild. However, recent studies of primate vocal communication suggest that number of syllables is a salient dimension for discriminating among conspecific calls. Ghazanfar, Flombaum, Miller, and Hauser (2001) demonstrated that cotton-top tamarin monkeys antiphonally call more in response to a complete, multi-syllabic species typical long call, compared with individual syllables from this call. They argue that in this context -- that is, in deciding whether or not to call back in response to another individual -- tamarins perceive only whole calls as a relevant unit. It is possible that they make this discrimination based on the number of syllables they expect to find in a complete call. With an acoustic stimulus like this, one can carefully control for dimensions other than number as one asks questions about the salience of number in categorizing these natural stimuli.

Gonzalez and Hauser (unpublished data) recently found that the same population of rhesus monkeys tested in this thesis consider copulation calls with more syllables to be more attractive. They played male calls numbering 5 or 2 syllables to estrous females and found that females reliably approached the speaker playing the 5-syllable calls. The researchers also controlled for variables other than number, such as overall duration of the calls and overall acoustic energy, and, therefore, effectively demonstrated a numerical discrimination between 2 and 5 for an acoustic stimulus. Clearly, this system holds tremendous potential with respect to studying number representation. First, this is a system in which on can ask about representations of number in the auditory domain using ecologically relevant stimuli. Second, it suggests that animals consider number important in a non-visual context. Numerous experiments are necessary to investigate whether the systems that animals use to represent number in a natural auditory context are similar to those implicated by the looking time experiments that I presented.

A recent neurophysiology experiment (Sawamura, Shima, and Tanji, 2002) found that monkeys of the same species have neurons in their parietal cortex that fire selectively when they need to carry out a fixed number of actions. Moreover, summing the firing rates of these neurons over the time required for these actions reveals a noisy signal of the kind implicated by the Accumulator model; firing for representations of 5 repeated actions are more noisy than firing for 2 repeated actions. These results not only begin to uncover the neural architectures potentially responsible for the numerical discriminations of Chapter 3, but they might uncover the architectures responsible for the natural behaviors observed by Gonzalez (unpublished data).

I conclude, therefore, optimistic about the paths that we are directed to by current research on spontaneous numerical representations in animals. We cannot yet distinguish between a number of alternative hypotheses with respect to the potential mechanisms that animals may use to solve problems dealing with numbers and quantity. Fortunately, however, the questions left unanswered are empirical. Using the type of comparative research paradigm I advocate in this thesis — one that compares across ages and species using the same spontaneous methods — we will soon be able to answer some of the most fundamental questions of human number representation. These answers will include pinning down the role of language, understanding the mechanisms of developmental change in humans, and, eventually, the neural underpinnings of human mathematics.