Chapter 1: Introduction

We have all had the experience of attending a carnival and participating in a raffle that requires us to guess the number of jellybeans in a full jar. Upon consideration, we realize that the clever raffle engineers have presented us with what is, in fact, a very difficult problem. Though we are all, generally speaking, excellent mathematicians with respect to our day-to-day needs – we easily buy groceries, mix ingredients in a recipe, and usually, balance our checkbooks – we are incapable of solving this simple counting problem precisely. Why? The answer, upon introspection at least, is that our mathematical abilities seem bound by our capacity to use and understand natural language. To make the point clear, imagine doing even a simple arithmetic problem such as balancing your checkbook. It is likely that you would approach such a problem by verbalizing it in your head in something like the following way: "forty-five dollars for groceries plus thirty on dry-cleaning comes out to seventy five; one-hundred in the account minus seventy-five leaves me with twenty-five." Even problems simpler than checkbook arithmetic seem to require the use of language. We cannot come to a definite conclusion about the number of jellybeans in a full jar because, to do so, we would need to serially attend to each bean and incrementally tag it with a number word. It appears, then, that we are great mathematicians in our day-to-day lives only by virtue of our linguistic abilities. If clever carnival owners limit our use of language, we fall apart mathematically.

Fortunately for us, situations with excessive linguistic constraints are limited to carnival games and the like. However, we can imagine that just as our regular activities demand knowledge of numbers, many other species of animals also encounter problems that require numerical insight on a regular basis. Determining the rate of energetic return from foraging in a particular food patch (Stephens and Krebs, 1986; Gallistel, 1990) is one such problem that many animals face, as is determining whether an intruding group is larger than one's own group (Hauser, 2000). While it is clear, therefore, that animals must possess some form of mathematical insight, it is less clear what form this insight might take. If our introspections are accurate, and we humans calculate numbers using language, then it is very difficult to imagine how animals might represent numbers. Number words, quantifiers, and number symbols provide human adults with precise information that can be applied easily and in a variety of contexts. By verbally counting the cookies in each jar, a five-year-old child could easily identify a jar with three cookies as more desirable than a jar with two cookies. It is not obvious, however, how a rhesus monkey would solve a comparable problem; for example, choosing to forage from a bush with more berries instead of from a bush with fewer berries.

Do animals solve problems like this in numerical ways? My goal in the following pages will be to answer this question by presenting some original data geared towards understanding whether or not language, or a mind that will develop language, is a necessary and/or sufficient prerequisite for certain types of numerical representations. To phrase this in the negative, can one have numerical representations without linguistic abilities? In this thesis, I will argue that, yes, one can represent numbers without language because not all numerical representations require a linguistic mind. I argue that there are at least two types of representation that animals use spontaneously when faced with problems that demand numerical insight, and I argue that even linguistically capable humans sometimes use these very representations. After all, someone at the carnival does always give a reply that is very near, if not exactly, the number of jellybeans in the full jar. They might call the process by which they arrive at their response “estimating,” “guessing,” “ball-parking,” or, sometimes, “guesstimating.” No matter what they call it, though, they claim to have some sense, albeit, imprecise, of how many jellybeans are in the jar. Indeed, we all have a qualitative sense of number as evidenced by the fact that everyone at a carnival makes guesses on the right order of magnitude; no one guesses 35 when the actual number is 213. This thesis will argue that just as humans at a carnival guesstimate readily and, actually, relatively accurately, nonhuman primates spontaneously guesstimate as well – perhaps even as well as humans do.

What counts as numerical?

Before I begin, however, there are a number of theoretical issues that require some clarification. The first of these is the issue of what it means to say that animals solve problems in “numerical ways.” An animal that performs a task numerically must represent number, not other properties of stimuli that might correlate with number, in order to solve ecologically or socially relevant problems. Returning to the example of choosing between a bush with more berries over a bush with fewer berries, non-numerical ways of solving this problem include estimating the amount of red (assuming that the berries are red) on each bush and then choosing the bush with more red. However, numerical ways of solving this problem are limited to methods that are sensitive to the addition or removal of some sub-set of berries from the bush, irrespective of those berries’ contribution to other properties of that bush. To be more specific, a numerical representation of the berries on a bush does not consider the extent to which a very small berry or a very large berry differentially contribute to the overall volume of the bush, the surface area of the bush, the amount of red on the bush, or the luminance of the bush, to name only a few examples. A numerical representation of the berries on a bush concerns only how many berries are on a bush. In more abstract terms, a numerical representation is sensitive to the addition or removal of individuals from a set, irrespective of those individuals' contribution to other properties of the set (Dantzig, 1967).

Returning to my previous discussion of language and number, as humans, it seems that linguistic labels might be critical to our understanding number as I defined it above – as a property of sets that is independent of every other continuous property. With this definition in mind, the main question in this thesis becomes, without language, could we ever understand that removing even one berry from a set results in a smaller set of berries? When trying to estimate how much red we see, a set of 5 berries might appear perceptually indistinguishable from a set of 6 berries. But, if we count each berry in these two groups, assigning the cardinal labels 5 and 6 to their respective sets gives us immediate information about which set is bigger, and by exactly how much. Further, in talking about 1 or 2 berries, we differentially apply the words "berry" and "berries." Quantifiers, a linguistic category present in nearly all known human languages (Hauser, 2000), intimate a sense of number (as opposed to other perceptual variables) when we are not even discussing numbers explicitly. Hence, although we can easily imagine non-numerical ways of choosing where to forage, specifically numerical ways of making such a decision seem unfathomable without the use of language.

The idea that humans rely on language to understand numbers has lead many investigators to ask not just about how animals represent number, but also about how preverbal human infants represent numbers, and about the types of mechanisms that support linguistic representations in human adults. Investigations into the psychological mechanisms used in numerical thinking developed within a rich comparative tradition that elucidates the development of numerical competence both within an individual's life span and over evolutionary time (Hauser and Carey, 1998). Asking questions about an animal's ability to understand numbers is interesting for two reasons. On the one hand, studies of animal competence tell us about the mechanisms available when animals solve problems in their environments, and the extent to which these mechanisms are shared across species. On the other hand, these studies also inform our understanding of the evolution of competence in humans. In particular, dissociations between human and nonhuman animal abilities identify the extent to which one species may have evolved a mechanism specialized for a particular domain, and also, the necessary and sufficient criteria to carry out certain cognitive tasks (Hauser and Carey, 1998). In the domain of number, for example, current evidence suggests that animals other than humans cannot represent large numbers precisely, though they can represent large numbers approximately (a more extensive discussion of these data is found below). Therefore, the ability to represent large numbers precisely might be a specialization unique to humans and may require a linguistic mind as a necessary and sufficient prerequisite.

An obvious problem with this line of reasoning, and one articulated extensively by Macphail (1994), is that dissociations observed between different animal species may be the result of methodological differences in testing and not of actual cognitive differences. Therefore, a strong comparative research program requires methods that can test a variety of species, including humans, with identical paradigms. In this thesis, I will describe several experiments with wild nonhuman primates focusing on the nature of number representation. These experiments have been designed to address questions of mechanism and evolution, by using the same methods with nonhuman primates and human infants. Specifically, these paradigms will allow me to ask if we share systems for non-linguistic number representation with our primate relatives. Further, because I will use methods that require little or no training, my thesis will inform questions about the strategies that animals in their natural environments use to solve problems requiring numerical insight. What follows, therefore, is a brief review of the literature on numerical abilities in animals and infants, as it pertains to the experiments presented later in this thesis. First, however, I address another theoretical concern that pertains to the type of research in this thesis.

Just numbers?

Before I examine the literature on number representation, I would like to consider whether it is reasonable to attempt to study number on its own, whether in monkeys, humans, or otherwise. In particular, given the relationship that numerical thought must have to other capacities – language, to name one example – does it make sense to design experiments that consider only number as a relevant parameter? This question gets at the core of more general theories that we might have about the architectures of the mind and the brain, and also, at the logic of the types of comparative experiments described in the following pages.

A common and older view of the mind (Piaget, 1952) suggests that we are endowed with a set of "all-purpose" computational tools that we can deploy to solve problems in a variety of contexts. More recent theories, however, and especially ones emerging within an evolutionary framework, argue that the human mind and brain are segregated into a number of relatively independent "tools" designed, by natural selection in many cases, for very specific tasks (Spelke, 1986; Fodor, 1983; Cosmides and Tooby, 1994; Hirschfeld and Gelman, 1994). In a computational system, such as the mind, specialization can be implemented in two major ways. First, at the processing, or input level, one can limit the data structures to which different computational components have access. What this means is that one can divide up computation (cognition) into different, relatively independent programs that are restricted to accessing certain databases. The theory that the mind is constructed in this way is known as the theory of modularity (Fodor, 1983). It suggests that independent computational units, or modules, in the mind are restricted with respect to the types of sensory information and remembered knowledge that they can make use of – they are informationally encapsulated.

A second way to specialize a computational system is by restricting the types of problems that engage it by restricting its available outputs. That is, a system could have the ability to produce only outputs relevant to a particular category of problems. A mechanism that is specialized in this way is known as domain-specific: it is specialized for producing outputs relevant to dealing with problems in a specific domain (Hirschfeld, and Gelman, 1994)

Extensive debates surround the topics of specialization in the mind and the brain, modularity, and domain-specificity (Flombaum, Santos, and Hauser, 2002). What is the relationship between these two types of specialization (Coltheart, 1999)? How is specialization implemented neurally (Scholl, 1997)? What is the role of natural selection in building systems with these two types of specialization (Cosmides and Tooby, 1994)? These are only a few among many questions that need to be asked and answered before a completely satisfying (and accurate) picture of the mind is rendered. Nevertheless, the large amount of interest and experimentation generated by discussions of modularity and domain-specificity have identified a number of signs indicating that a system is reasonably characterized by one, if not both, of these terms. Specialized cognitive mechanisms, especially modular ones, are often fast, automatic, mandatory, susceptible to selective impairment, and localized to very specific neural areas (Fodor, 1983; Flombaum, Santos, and Hauser, 2002). Moreover, all of these characteristics suggest that a system is relatively independent.

Current evidence suggests that the systems sub-serving numerical cognition can be aptly characterized in some of these ways. Below, I extensively discuss looking time experiments that demonstrate that infants and nonhuman primates with no prior training or experience are sensitive to inconsistencies in the outcomes of simple arithmetic operations with real objects. They generate the representations necessary for manifesting this sensitivity quickly and automatically under experimental conditions. An extensive literature on animal number representations using operant conditioning has lead to the suggestion that animals can represent number with analog, magnitude-like representations. This hypothesis will receive extensive attention below and in Chapter 3, but it should be noted here that representations of this type are stereotypically domain-specific. Mental magnitudes, while imprecise and not useful in solving number problems that require precise knowledge, are probably sufficient for solving many of the numerical problems that animals regularly face. The animal number system is, therefore, domain-specific as only representations of a very specific format can be generated to solve problems in the domain of number. Finally, Dehaene (1997) reviews the many cases of a striking disorder known as acalculia in which patients with selective brain damage, usually to parietal regions, suffer an inability to perform many numerical tasks despite conserving more general cognitive abilities, including language. Collectively, these data lead him to conclude that humans share with many other species of animal a very specialized capacity for understanding number that he calls a "number sense."

If nonhuman animals and humans share some specialized and independent abilities in the domain of number, and we can tap these abilities through controlled experiments, then what should we make of instances where one species does not perform as well as another? In particular, we expect that some numerical abilities should be unique to adult humans that have a capacity for natural language. When animals fail at tasks that humans pass, can we argue that these tasks require a mind with language, or, at least, a mind that will develop language? As mentioned above, Macphail (1994) cautions against drawing conclusions about the cognitive prerequisites for certain abilities from animal failures in tasks that we think require these abilities. More specifically, he draws a distinction between the ability to perform some task, and one's performance in this task. Often, it is the case that one is able to do something, though this might not be obvious in one's performance. Perhaps task demands not directly relevant to the mechanisms of interests confound a species' performance in cases like these.

Recently, a research program has been developed that is directly interested in tracing the evolution of human cognitive capacities by comparing dissociations in abilities between preverbal human infants and nonhuman primates (Hauser and Carey, 1998; Hauser, 1996). Precisely to avoid the problems addressed by Macphail (1994), this program employs identical methods for testing infants and monkeys. In this way, one can interpret dissociations in performance as reflecting ability. The literature that I discuss below, especially as it pertains to the Object File model of small number representation, focuses on experiments that implement identical methods with animals and humans.

A brief review of number research

A. *The Accumulator*

There are two ways to consider experiments concerning the abilities of animals in the domain of number. One can ask (1) what are the spontaneous abilities of animals in the number domain? And (2) what are the abilities of animals to represent number and use computations over such representations? The distinction between these two questions is that answering the second may identify things that an animal can learn, while only answering the first necessarily tells us about the tools that an animal can and might use in the wild. Many experiments that involve extensive training and operant conditioning pertain only to the second question. These methods, while addressing what animals *can do*, do not, necessarily, inform studies of what animals *actually do* when faced with problems in the wild (Hauser, MacNeilage, and Ware, 1996; Hauser, Carey, and Hauser, 2000). A major triumph of these studies, however, is the discovery of particular properties common to the non-linguistic representations of number used by animals, human infants, and, even, human adults in certain contexts (Gallistel and Gelman, 2000 for review). This convergence among experiments testing a variety of organisms suggests that these experiments do, indeed, identify mechanisms available to an animal spontaneously, and it has led to the development of one model of non-linguistic number representation known as the Accumulator model (Meck and Church, 1983).

The Accumulator model proposes that non-linguistic number representations come in the form of noisy magnitudes. Therefore, animals represent numerosity approximately instead of exactly, but can nevertheless perform operations over these representations including addition, subtraction, division, and multiplication (Gallistel and Gelman, 2000). Platt and Johnson (1971) trained rats to press an armed feeder after making a fixed number of lever presses. They found that the rats pressed with relatively high precision for small numbers, with variation increasing as number increased. More interestingly, while the mean response matched the target number of presses, the coefficient of variation around this mean remained constant. That is, the variance increased in proportion to the mean, and therefore, in proportion to the required number of presses. This effect is known as scalar variability, and follows from the Accumulator model because as numbers become larger, their magnitude representations incorporate more noise at regular intervals.

Scalar variability in the Accumulator model predicts not just a greater difficulty in representing larger numbers -- sometimes called a “size effect” -- but also, greater difficulty discriminating between two numbers with a small numerical distance between them – sometimes called a “distance effect.” These size and distance effects are prevalent in studies of numerical abilities in animals, and they support the hypothesis that animals can represent numbers with approximate mental magnitudes. Brannon and Terrace (2000), for example, trained two rhesus monkeys to order visual arrays of 1-4. They showed that after extensive training on this task, the animals "spontaneously" generalized what they had learned and could correctly order pairs of cards with arrays between 1 and 9 despite having never been trained with arrays containing numbers between 5 and 9. As predicted by the Accumulator model, however, the animals' abilities to represent the numbers in these arrays were not precise, demonstrating both a size and distance effect; subjects made more errors with larger numbers, and with two close numbers (e.g. 4 and 5) compared to two far numbers (e. g. 4 and 8). These kinds of results have also been found in similar studies with chimpanzees (Tomonaga and Matsuzawa, 2000) as well as pigeons (Emmerton, Loheman, and Niemann, 1997). Likewise, animals that have been taught a symbolic number system – Arabic numerals or number words, for example – have also shown scalar variability when asked to compare number symbols or to use them to identify the numerosity of a set (Pepperberg, 1994 for experiments with a parrot; Boysen, 1993, Matsuzawa, 1985, and Thomas, 1992 for experiments with chimpanzees).

What conclusions can be drawn from the above experiments? These experiments suggest that animals can learn to represent small as well as large numbers approximately by using Accumulator-like magnitudes. However, these experiments do not demonstrate that animals spontaneously use number representations in their natural environments. Despite the fact that in some experiments (Branon and Terrace, 1998) animals will generalize a rule learned for small numbers to larger numbers, the extensive training required for small numbers limits interpreting these experiments as demonstrations of what animals actually do spontaneously.

In fact, very few experiments have addressed the spontaneous abilities of animals with respect to number. This is primarily because without a paradigm that involves training, it is very difficult to be sure that animals discriminate based upon number, and not a property that correlates with number such as duration or over-all amount of "stuff." Rumbaugh, Savage-Rumbaugh, and Hegel (1987) gave chimpanzees a choice between two trays with varying numbers of candy pieces. Each of the two trays had two wells, and, therefore, in order to select the tray with more candy, the chimpanzees needed to "add" the number of pieces in each well. They argue, based on their results, that chimpanzees can spontaneously use approximate, magnitude representations of number, and, also, that they can "add" these representations. However, because all the candy pieces were in view during this experiment, it is unclear whether the chimpanzees made their selections in response to the number of candies on each tray, or, in response to a low-level perceptual assessment of the overall quantity or continuous extent in each tray.

Few experiments with animals provide direct evidence with respect to spontaneous number representations. However, experiments with adult humans as well as with human infants have done a more convincing job of demonstrating the spontaneous use of magnitude-like number representations. In one experiment (Whalen, Gallistel, and Gelman, 1999) college students preformed a task very similar to the one employed by Platt and Johnson (1971) in studying rats. Here, Arabic numerals between 9 and 25 appeared on a computer screen, and the participants were instructed to tap a key as quickly as they could, without counting, to match the number represented by the

Arabic numerals. Inter-tap durations were faster than the speed of sub-vocal counting, and the authors argued that these tap data were the product of a nonverbal counting process. Moreover, they found that, like the rats (Platt and Johnson, 1971), humans made more errors with larger numbers, but that the coefficient of variation around the correct (and mean) number of presses remained more or less constant. Based on these data, they conclude that humans share with other animals a capacity for representing number with approximate mental magnitudes.

An earlier experiment (Moyer and Landauer, 1967) similarly found scalar variability in human number representations even when subjects needed only to compare Arabic numerals. Participants in this experiment could more quickly identify the larger number in a pair when the numbers compared were relatively small, and when the distance between the two numbers were large. For example, comparing the numbers 2 and 5 is easier than comparing the numbers 6 and 5. Moyer and Landauer (1967) concluded, from these data, that noisy mental magnitude representations underlie humans' ability represent number with precise numerals.

Interested, in part, in the seeming relationship between linguistic number representations and magnitude representations in adults, a number of developmental researchers have asked if preverbal human infants can represent number as well, and if they do so with mental magnitudes (Wynn, 1992; Xu and Spelke, 2000; Simon, Hespos, and Rochat 1997, to mention only a handful). Xu and Spelke (2000) tested six-month-old infants, asking if after habituation to arrays of 8 dots they dishabituated upon seeing arrays of 12 or 16 dots. They found that when all the relevant confounding variables were controlled (e. g. contour, spatial extent, luminance) across the displays, the infants dishabituated during a transition from 8 to 16 dots (or vice-versa) but not during a transition from 8 to 12 dots (or vice-versa). They argue that the infants in this experiment use Accumulator-like mental magnitudes to represent the numbers in these arrays because, according to the Accumulator model, numbers that differ by a large ratio of 1:2, such as 8 and 16, are readily discriminated, while numbers differing by a smaller ratio of 2:3 (8 and 12) are more difficult to discriminate.

Collectively, the experiments described above suggest that Accumulator-like magnitude representations of number are shared among human adults, human infants, and other animals including nonhuman primates. These experiments suggest that analog magnitude representations may underlie the use of symbolic representations by humans, and they also demonstrate that human adults and infants spontaneously deploy magnitude representations in certain contexts. With respect to the spontaneous abilities of animals, though, there are fewer experiments. These few experiments, which I discuss extensively below, focus primarily on representations of small numbers, and they implicate a system different from the Accumulator. As of yet, there is no evidence that animals spontaneously represent large numbers, or that animals spontaneously represent number with noisy mental magnitudes.

B. Indexing and Object Files

Studies with preverbal human infants have done a more convincing job of illustrating how number could be spontaneously represented without language. These investigations have lead to a second model of numerical representation applicable, however, only to small numbers of things. Wynn (1992) used the violation of expectancy paradigm to demonstrate that infants expect the outcome of a 1+1 event to be exactly 2, and the outcome of a 2-1 event to be exactly 1. In this paradigm, looking time is measured while subjects observe items being placed behind an occluder. When the occluder is removed, in some conditions, subjects find the expected number of items, while in others, they find an unexpected number of items. The logic of the paradigm is that if infants detect a violation, they will look longer at such events because they are unexpected or "surprising." In Wynn's (1992) experiment, four-and-a-half month old infants observed a Mickey Mouse doll on a stage. Wynn then placed a screen in front of the doll, and then placed another Mickey Mouse doll behind the screen as the infants watched. When the screen was removed, subjects saw either 2 dolls on the stage – the expected condition – or, 1 or 3 dolls on the stage – the unexpected conditions. She found that infants looked longer at both the unexpected conditions compared to the expected condition indicating that they spontaneously represented the outcome of the 1+1 event as exactly 2.

Hauser and colleagues (1996) adapted Wynn's (1992) methods for use with semi-free ranging rhesus monkeys. With eggplants instead of Mickey Mouse dolls, they demonstrated that monkeys also expect 1+1 to be 2. Similarly, the monkeys in this study looked longer if the outcome of a 2-1 event was something other than 1. Therefore, like the infants in Wynn's (1992) study, rhesus monkeys spontaneously represent small numbers in the range of 1-3. Although these studies provide a replication of the infant studies with a nonhuman primate, it should be noted that not all studies have replicated Wynn's (1992) results, and, more importantly, when spatial extent and contour length are properly controlled (contra Wynn, 1992), infants fail at these same tasks (Clearfield and Mix, 1999; Feigenson, Carey, and Spelke discussed extensively in Chapters 3 and 4).

Uller, Hauser, and Carey (2001) were not satisfied with the argument that these results indicate that infants and monkeys represent the number of items in these looking time events. They argue that the subjects in these experiments might simply represent the amount of "stuff" that they expect to see behind an occluder instead of the actual number of items that they should find there. To test this possibility, they first established that a group of captive cotton-top tamarin monkeys (hereafter tamarins) also expect the outcome of a 1+1 event to be 2, not 1 or 3. However, as an added unexpected condition, they presented the tamarins with one large piece of food that was twice the size of the original piece. The tamarins looked longer in this condition as well, suggesting that they might represent the number of items they expect to find behind the screen; still, other controls are necessary, both here and in the infant studies.

While the three looking time experiments described above might demonstrate that preverbal infants as well as nonhuman primates spontaneously represent the number of a set smaller than 3 or 4, these experiments do not demonstrate exactly how this is done. Do the participants in these experiments deploy approximate mental magnitudes to solve the problems that they face? If they do make use of approximate mental magnitudes in looking time experiments, then one expects a Weber-fraction limit upon the ratio between the expected and unexpected outcomes that are discriminated. For example, because the infants in Wynn's (1992) study discriminate between an expected outcome of 2 and an unexpected outcome of 3, they should also discriminate between other outcomes that share a 2:3 ratio, such as an expected outcome of 4 and an unexpected outcome of 6 in viewing a 2+2 event. However, they should not, in viewing such events, discriminate between an outcome of 4 and an outcome of 5 because the 4:5 ratio is too small to be discriminable. Thus, hypothesizing that an individual uses mental magnitudes in solving a problem makes very specific predictions about the types of numbers that this individual should be able to discriminate. This is one approach that one can take in determining how infants and animals represent number in looking time experiments. I will present an experiment with such an approach in Chapter 3, but it is worth noting that such a systematic cataloging of discriminable ratios has not yet been done with human infants to determine what type of representations support performance in experiments such as Wynn (1992).

Uller, Carey, Huntley-Fenner, and Klatt (1999) used a different approach to investigate the types of representations that might provide a basis for the successes of infants in looking time experiments. They hypothesized, based on work with human adults (Pylyshyn and Storm, 1988; Trick and Pylyshyn, 1994; Kahneman, Treisman, and Gibbs, 1992), that the infant visual system can spontaneously and in parallel individuate up to four objects. Extensive evidence suggests that upon first encountering a visual scene or display, the visual system picks out a small number of independent objects, assigns them with indices trackable in time and space, and constructs simplified, imagistic representations of each individuated object, sometimes called "Object File representations." Uller and colleagues (1999) observed that if the infant visual system does the same, then infants have at their disposal a mechanism useful for enumerating small sets of visual objects. That is, infants could solve looking time problems like those presented in Wynn (1992) by constructing an Object File representation – like opening a drawer in a filing cabinet – for each object placed behind the screen. When the screen in these experiments is removed to display an expected or an unexpected outcome, the infants can detect violations by performing a correspondence operation between the discrete Object Files "open" in working memory, and the objects present on the stage.

There are a number of important differences between the Object File and the Accumulator hypothesis, and I will discuss them more extensively below. Uller and her colleagues (1999), however, capitalized on one difference in particular in order to determine how infants might represent the outcome of a "1+1" event in a looking time experiment. They argue that the Object File system is susceptible to more error when a numerical task places memory demands upon a subject compared to when it does not. This is the case, they claim, even if the two tasks in questions deal with an identical problem for all numerical purposes. This, they argue, is because the construction of Object File representations and the process of keeping them in mind require the use of working memory. Moreover, they argue that this is not the case for Accumulator representations. For the Accumulator model, which constructs only one final representation to represent the cardinal value of a set, a number is a number is a number, so to speak. To represent the number "4" for example, the Accumulator model will construct a single magnitude representation that places no more of a load upon memory than a single magnitude representation of the number "2." With the Object File model, however, representing the number "4" requires that subjects construct and maintain in working memory two more representations than for the number "2." This places more severe demands upon working memory. If infants use Object Files then they should demonstrate differential success at representing identical events that, somehow, place differential demands upon working memory. The Accumulator model makes no such prediction, and, instead, predicts that representing the outcome of a 1+1 event should be equally easy under all conditions.

Experimental tests of these contrasting predictions support the hypothesis that infants use Object Files, and not mental magnitudes, to represent small numbers in arithmetic events (Uller et al., 1999). 8-month old infants failed to discriminate between correct and incorrect outcomes in a 1+1 event if both objects were placed behind a screen on a stage. They could, however, discriminate between outcomes in viewing these events if one object was placed on the stage before the screen was put in place. The authors of this study argue that constructing an Object File representation of a visible object, such as one sitting atop a stage, is easier than constructing such a representation of an object that disappears from sight, such as one that is placed behind a screen. In the “screen-first” condition that the 8-month olds in this experiment fail, they need to construct two representations of objects that disappear from sight. Nothing is present on the stage at the beginning of this condition, and then the infants must update their representation of nothing on the stage with a representation of two objects that are placed onto the stage behind the screen. However, In the “object-first” condition that these infants pass, and that 5-month olds pass in other experiments (Wynn, 1992), an Object File representation is constructed for one visible object – the on placed on the stage first – and for only one invisible object – the one that is added behind the screen. The additional working memory requirements associated with representing objects placed on the stage behind the screen compared to objects placed on the stage before the screen’s implementation, Uller and colleagues (1999) argue, accounts for the inability of 8-month old infants to discriminate in the “screen-first” condition compared to the “object-first” condition.

Further, they point out that 10-month old infants, because of a developmentally contingent increase in working memory capacity, do discriminate between outcomes in both versions of the 1+1 events. This difference between screen and object first conditions and between 8-month olds and 10-month olds, Uller and colleagues (1999) argue, is consistent with the Object File model but not with the Accumulator model of number representation.

Beyond placing variable demands upon working memory, the Accumulator and the Object File models differ from one another in a number of other important ways. The Object File model provides each to-be-counted item in a set with a discrete representation of its own, whereas the Accumulator model constructs only one representation that functions for all the items in a set collectively. However, whereas there is seemingly no end to the number of items that the Accumulator model can represent, the Object File model is limited by the number of Object Files available. In experiments with adults asked to track a subset of items within an array of identical items (Storm and Pylyshyn, 1988), and experiments in which adults are asked to quickly identify the number of items in a briefly presented visual display (Trick and Pylyshyn, 1994), parallel individuation is limited to 4 objects at once, suggesting that the visual system is limited to 4 Object Files. Therefore, an organism that uses Object Files to discriminate between different numbers of things should be limited to succeeding with only 4 or maybe 5 items (if the organism can represent "more than the number of Object Files I have"). However, within this small number range, all differences and ratios between numbers should be as readily discriminated. As discussed above, the Accumulator model has a different set of limitations. The Accumulator is able to represent both small and large numbers of things, though an organism using this system should only discriminate between sets that differ by large ratios. Thus, a set-size signature of 1-4 characterizes the Object File system, while a Weber-fraction limit upon discrimination characterizes the Accumulator model.

Hauser, Carey, and Hauser (2000), and Feigenson, Carey, and Hauser (2002) designed a second paradigm to investigate the number representations used by primates and infants. These studies were designed to test for convergent evidence of number representation using a search as opposed to a looking time task, and they were designed to investigate the types of representations used by these organisms by considering the unique characteristics of the models described above. In their studies two empty boxes were placed to the left and to the right of an experimenter, and equidistant from a rhesus monkey or an infant. The subjects then watched as the experimenter placed pieces of food into the boxes. When the experimenter stepped away, subjects could take the food in one of the boxes, though the boxes were too far apart from each other for the subject to obtain the food from both boxes. In this way, these investigators ask if the infants and the monkeys discriminate between the different numbers of food items in the two boxes. Monkeys and infants, they hypothesized, would always choose the box with more food so long as they understood that one box had more food than another. They found that both infants and rhesus monkeys consistently chose a box with 2 pieces of food over a box with 1 piece of food, as well as a box with 3 pieces of food over a box with 2 pieces. The infants, however, choose a box with 4 pieces equally as often as they chose a box with 3, though the monkeys consistently chose 4 over 3. Interestingly, neither the monkeys nor the infants could successfully discriminate 3 from 6 pieces of food. Based upon these results, the authors of these studies argue that both infants and rhesus monkeys can spontaneously represent small numbers for use in a task that mimics a real foraging problem. Further, they argue that the subjects in these experiments do not use mental magnitudes to represent number. The Accumulator model predicts that the ease with which one can discriminate between two numerosities should depend upon the ratio between them. In these experiments, the subjects successfully discriminate 1 and 2, but not numbers differing by the same 1:2 ratio – 2 v. 4 and 3 v.6 for the infants, and 4 v. 8 in the case of the rhesus. Instead, these investigators argue that the system used by the infants and the monkeys to represent number is limited to the numbers 1-4. They propose that the subjects in these experiments represent these numbers with Object Files. The Object File model predicts a limit of approximately 4 with respect to parallel individuation, a limit that appears to characterize adult humans (Trick and Pylyshyn, 1994) as well as nonhuman primates. This fact, according to these researchers, accounts for the limit upon the numbers that the subjects in these experiments successfully discriminate.

Summary and aims

In this chapter, I provided a general introduction into the conceptual and methodological issues associated with the problem of number representation in animals. It should be clear that in many different training contexts animals seem to share with humans an ability to represent numbers approximately with analog mental magnitudes. It should also be clear that very little data exists with respect to the spontaneous abilities of animals, especially, in the domain of large numbers. Because data in this area is lacking, there is also no evidence, to my knowledge, suggesting that animals spontaneously represent large numbers with analog, Accumulator-like mental magnitudes.

In Chapter 3 I will present some new experiments run with the population of semi-free ranging rhesus monkeys tested by Hauser, MacNeilage, and Ware (1996), and by Hauser, Carey, and Hauser (2000). I selected this population for three main reasons. First, the monkeys tested in the following experiments range freely over a small island, and this allows for their testing in a relatively naturalistic environment. These monkeys also make good candidates for the study of number because Branon and Terrace (1998) demonstrated that monkeys of the same species could learn ordinal relationships among arrays in a laboratory setting. Finally, rhesus monkeys have long been a common animal subject in neurophysiology experiments and other studies of brain function. Therefore, the potential exists to extrapolate from results in behavioral experiments to the design of experiments that investigate the neural substrates of numerical representation.

To test these monkeys, I will employ looking time methods in experiments designed to address one major question in the area of number representation: if and how do monkeys spontaneously represent large numbers? Before presenting these new experiments, however, I will proceed to offer a more thorough description of this study population and the methods employed in my experiments.